

chain-consistent-updates1^{13,45}

$$\begin{aligned} & \forall es:ES, Cmd:Type, In:AbsInterface(Cmd), isupdate:(Cmd \rightarrow \mathbb{B}), Sys, Out:AbsInterface(Top). \\ & (E(In) \subseteq_r E(Sys)) \\ & \Rightarrow (E(Out) \subseteq_r E(Sys)) \\ & \Rightarrow (\forall e:E(In). (\neg(\uparrow(isupdate(In(e))))) \Rightarrow (\uparrow(e \in_b Out))) \\ & \Rightarrow (\forall f:sys\text{-antecedent}(es;Sys). \\ & \quad (\forall u:E(Sys). (f(u) = u \in E) \Rightarrow (\uparrow(u \in_b In))) \\ & \Rightarrow (\forall a, b:E(In). \\ & \quad \text{effective}(a) \\ & \quad \Rightarrow \text{effective}(b) \\ & \quad \Rightarrow (\uparrow(isupdate(In(a)))) \\ & \quad \Rightarrow (\uparrow(isupdate(In(b)))) \\ & \quad \Rightarrow ((a \in \text{cr-explanation}\{i:l\} \\ & \quad \quad \quad (es; Sys; f; b)) \\ & \quad \quad \vee (b \in \text{cr-explanation}\{i:l\} \\ & \quad \quad \quad (es; Sys; f; a)))) \\ & \Rightarrow \text{consistent-updates}(es;In;Out;Cmd;isupdate;\lambda e.\text{cr-explanation}\{i:l\}(es; Sys; f; e)) \end{aligned}$$